

“Algebra Universale- Teoria dei modelli”
(Scuola Estiva di Logica- 2008-Aldo Ursini)
6 lezioni (2 ore ciascuna)+ esercitazioni

Content of the course: *An introduction to Universal Algebra.*

The aim is to introduce some of the basic tools and results of universal algebra with connections to model theory and the algebraization of logics.

Part 1.(~3 hours)

Partial orders and Lattices (modular, distributive, complete, algebraic)

Basic notions on algebras. Construction of algebras: subalgebras; generation of subalgebras; direct products; subdirect products; ultraproducts; congruences, quotients and homomorphisms.

Recalling a few notions of first order logic: structures, interpretations. Strong Completeness Theorem. Compactness Theorem. Lemma on Ultraproducts.

Part 2.(~4 hours)

Subdirectly irreducibles: Birkhoff's Theorem and applications.

Term algebras.

Equational Classes, Free Algebras and Varieties: Birkhoff's Theorem

Equational logic: Completeness Theorem.

Finite Basis for identities. Decidability of Equational theories.

Quasivarieties: Mal'cev's Theorem

Part 3.(~2 hours)

Mal'cev Conditions on varieties : permutable congruences; modular congruences ; distributive congruences; subtractive varieties.

Part 4.(~3 hours)

Algebraizations of Logics: Boolean and Heyting algebras. General algebraizable logics.

Reading suggestions:

❖ As a basic reference, we take

Burris-Sankappanavar, *A course in Universal Algebra*,

which is available online at: <http://www.math.uwaterloo.ca/~snburris/htdocs/ualg.html>

This covers parts 1-2-3 of the program (and of course contains more than we will be able to mention in the lectures) in Chapters I and II, in Sec.1-6 of Chapter IV, and in Chapter V.

We recommend a careful reading of the “Preliminaries” (pages “1,2,3”) where basic notations and facts from set theory are summarized and will be used in the lectures.

For part 4, a reference is:

Blok W.J., Pigozzi, D. *Algebraizable Logics*, Am.Math. Soc. Memoirs, N. 396, 1989

❖ For at least some of the topics, an handout of lecture notes will be available before the Summer school begins.

❖ Other bibliographic references will be indicated during the lectures.

❖ The site <http://spot.colorado.edu/~kearnes/ua.html> contains links to several online sources for Universal Algebra.