

Prediction, learning, and games

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Errata (September 8, 2006)

Pages 12–13, polynomially weighted average forecaster. Theorem 2.1 cannot be invoked to prove Corollary 2.1 because the polynomial potential $\Phi_p(\mathbf{u}) = \|\mathbf{u}_+\|_p^2$ is not twice differentiable when \mathbf{u} has one or more components equal to 0 (thanks to Amy Greenwald and Casey Marks for pointing this out). However, we can use Theorem 2.1 to prove the following slightly weaker version of Corollary 2.1 in which the factor $\sqrt{p-1}$ has been replaced by \sqrt{p} .

Corollary 1 *Assume that the loss function ℓ is convex in its first argument and that it takes values in $[0, 1]$. Then, for any sequence $y_1, y_2, \dots \in \mathcal{Y}$ of outcomes and for any $n \geq 1$, the regret of the polynomially weighted average forecaster satisfies*

$$\hat{L}_n - \min_{i=1, \dots, N} L_{i,n} \leq \sqrt{npN^{2/p}}.$$

Proof. We make the following observation

$$\begin{aligned} \Phi_p(\mathbf{R}_n) &= \left(\sum_{i=1}^N (R_{i,n})_+^p \right)^{2/p} \leq \left(\sum_{i: R_{i,n} \leq 1} (R_{i,n})_+^p + \sum_{i: R_{i,n} > 1} (R_{i,n})_+^p \right)^{2/p} \\ &\leq \left(N + \sum_{i: R_{i,n} > 1} (R_{i,n})_+^p \right)^{2/p} \leq N^{2/p} + \left(\sum_{i: R_{i,n} > 1} (R_{i,n})_+^p \right)^{2/p} \end{aligned}$$

where in the last step we used the fact that $(a+b)^c \leq a^c + b^c$ for $a, b \geq 0$ and $0 \leq c \leq 1$ (recall that $p \geq 2$ implying $0 \leq 2/p \leq 1$). Let $\mathcal{I} = \{i : R_{i,n} > 1\}$. Because of the boundedness of the loss function, we have $R_{i,n-1} > 0$ for each $i \in \mathcal{I}$. Since both vectors

$(R_{i,n-1})_{i \in \mathcal{I}}$ and $(R_{i,n})_{i \in \mathcal{I}}$ lie in the positive orthant of $\mathbb{R}^{|\mathcal{I}|}$, we can apply Theorem 2.1 and obtain

$$\left(\sum_{i \in \mathcal{I}} (R_{i,n})_+^p \right)^{2/p} \leq \left(\sum_{i \in \mathcal{I}} (R_{i,n-1})_+^p \right)^{2/p} + (p-1)|\mathcal{I}|^{2/p} \leq \Phi_p(\mathbf{R}_{n-1}) + (p-1)N^{2/p}$$

where we used again the boundedness of the loss. We can then write

$$\Phi_p(\mathbf{R}_n) \leq N^{2/p} + \Phi_p(\mathbf{R}_{n-1}) + (p-1)N^{2/p} \leq \Phi_p(\mathbf{R}_{n-1}) + pN^{2/p}.$$

Iterating this argument n times gives $\Phi_p(\mathbf{R}_n) \leq pnN^{2/p}$. The proof is then concluded in the same way as the proof of Corollary 2.1.

Page 41, Lemma 3.1. The statement of Lemma 3.1 should read as follows:

For each $t = 1, 2, \dots$ let $E_t^* = \operatorname{argmin}_{E \in \mathcal{E}} \sum_{s=1}^t \ell(f_{E,s}, \mathbf{y}_s)$. Then for any sequence $\mathbf{y}_1, \dots, \mathbf{y}_n$ of outcomes,

$$\sum_{t=1}^n \ell(\mathbf{p}_t^*, \mathbf{y}_t) \leq \sum_{t=1}^n \ell(f_{E_t^*, t}, \mathbf{y}_t) = \inf_{E \in \mathcal{E}} L_{E,n}.$$

The notation in the proof should be changed accordingly. (Pointed out by Giovanni Cavallanti.)

Page 274, Exercise 9.16. “Markov” with capital M.

Page 328, lines 2–3. “Minimization the Kullback-Leibler divergence” should be replaced by “Minimization of the Kullback-Leibler divergence”

Page 336, line -14. Missing “)”

Page 339, Proof of Theorem 12.2. Replace the first two sentences in the proof by the following:

Recall the normalized exponential potential update $w_{i,t} = w_{i,t-1} e^{-z_i} / \left(\sum_{j=1}^d w_{j,t-1} e^{-z_j} \right)$ for $i = 1, \dots, d$, where $\mathbf{z} = \lambda \nabla \ell_{\gamma,t}(\mathbf{w}_{t-1})$. By combining the argument that leads to (12.1) with the proof of Theorem 11.3 we obtain

$$\begin{aligned} \lambda(\gamma - \ell_{\gamma,t}(\mathbf{u})) \mathbb{I}_{\{\hat{y}_t \neq y_t\}} &\leq \lambda(\mathbf{u} - \mathbf{w}_{t-1}) \cdot (-\mathbf{z}) \\ &= D_{\Phi^*}(\mathbf{u}, \mathbf{w}_{t-1}) - D_{\Phi^*}(\mathbf{u}, \mathbf{w}_t) + \ln \left(\sum_{j=1}^d w_{j,t-1} \exp(\mathbf{w}_{t-1} \cdot \mathbf{z} - z_j) \right) \\ &\leq D_{\Phi^*}(\mathbf{u}, \mathbf{w}_{t-1}) - D_{\Phi^*}(\mathbf{u}, \mathbf{w}_t) + \frac{\lambda^2}{2} X_\infty^2 \end{aligned}$$

where in the last step we have applied Hoeffding inequality (Lemma A.1) to the random variable with range $\{z_1, \dots, z_d\} \subseteq [-X_\infty, X_\infty]$ and distribution \mathbf{w}_{t-1} .

Page 344, boxed figure. Two consecutive steps have the same label “(2)”

Page 344, statement of Theorem 12.4. The sentence “Then the number of mistakes $m = \sum_{t=1}^{\infty} \mathbb{I}_{\{\hat{y}_t \neq y'_t\}}$ is finite and satisfies...” should be replaced by “Then m is finite and satisfies...”.